**Exercise 24.1-1**

Run the Bellman-Ford algorithm on the directed graph below. Use vertex $z$ as the destination and illustrate how $first$ changes throughout the execution. Then change the weight of edge $(z, x)$ to 4 and run the algorithm again, using $s$ as the destination. *(Note: the problem statement has been changed to be consistent with the lecture notes.)*

![Graph Image](image)

**Solution:**

(a) Use vertex $z$ as the destination.

From the figure above we know:

- $w(s, t) = 6$, $w(s, y) = 7$, $w(t, y) = 8$, $w(t, x) = 5$, $w(t, z) = -4$, $w(x, t) = -2$, $w(y, x) = -3$, $w(y, z) = 9$.

$i = 1$:

Only need to consider edges ending in $z$; rest are $\infty + ? = \infty$.

- $(t, z)$ -4; $(y, z)$ 9

$i = 2$:

For $s$, check $M[1, t] + 6 = 2$; $M[1, y] + 7 = 16$.

For $t$, check $M[1, y] + 8 = 17$; $M[1, x] + 5 = \infty$; $M[1, z] + (-4) = -4$.

For $x$, check $M[1, t] + (-2) = -6$.

For $y$, check $M[1, x] + (-3) = \infty$; $M[1, z] + 9 = 9$.

$i = 3$:

For $s$, check $M[2, t] + 6 = 2$; $M[2, y] + 7 = 16$.

For $t$, check $M[2, y] + 8 = 17$; $M[2, x] + 5 = -1$; $M[2, z] + (-4) = -4$.

For $x$, check $M[2, t] + (-2) = -6$.

For $y$, check $M[2, x] + (-3) = -9$; $M[2, z] + 9 = 9$.

$i = 4$:

For $s$, check $M[3, t] + 6 = 2$; $M[3, y] + 7 = -2$.

For $t$, check $M[3, y] + 8 = -1$; $M[3, x] + 5 = -1$; $M[3, z] + (-4) = -4$.

For $x$, check $M[3, t] + (-2) = -6$.

For $y$, check $M[3, x] + (-3) = -9$; $M[3, z] + 9 = 9$.

<table>
<thead>
<tr>
<th>$M$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$s$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>2</td>
<td>2</td>
<td>$\infty$</td>
</tr>
<tr>
<td>$t$</td>
<td>$\infty$</td>
<td>-4</td>
<td>-4</td>
<td>-4</td>
<td>-4</td>
</tr>
<tr>
<td>$x$</td>
<td>$\infty$</td>
<td>9</td>
<td>$\infty$</td>
<td>-6</td>
<td>-6</td>
</tr>
<tr>
<td>$y$</td>
<td>$\infty$</td>
<td>9</td>
<td>-6</td>
<td>$\infty$</td>
<td>-9</td>
</tr>
</tbody>
</table>
From the above table we can find that the shortest path from s to z is: s → t → z

Shortest path from t to z is: t → z
Shortest path from x to z is: x → t → z
Shortest path from y to z is: y → x → t → z

(b) Set weight of edge (z, x) to 4 and use vertex s as the destination.

\[ w(t, y) = 8, w(t, z) = 5, w(x, t) = -4, w(x, z) = -2, w(y, x) = -3, w(y, z) = 9, w(z, x) = 4, w(z, s) = 2. \]

\[ i = 1: \]

Only need to consider edges ending in s; rest are \( \infty + ? = \infty \).

\((z, s) \rightarrow 2\)

\[ i = 2: \]

For t, check \( M[1, y] + 8 = \infty; \ M[1, x] + 5 = \infty; \ M[1, z] + (-4) = -2. \)
For x, check \( M[1, t] + (-2) = \infty. \)
For y, check \( M[1, x] + (-3) = \infty; \ M[1, z] + 9 = 11. \)
For z, check \( M[1, x] + 4 = \infty; \ M[1, s] + 2 = 2. \)

\[ i = 3: \]

For t, check \( M[2, y] + 8 = 19; \ M[2, x] + 5 = \infty; \ M[2, z] + (-4) = -2. \)
For x, check \( M[2, t] + (-3) = -4. \)
For y, check \( M[2, x] + (-3) = \infty; \ M[2, z] + 9 = 11. \)
For z, check \( M[2, x] + 4 = \infty; \ M[2, s] + 2 = 2. \)

\[ i = 4: \]

For t, check \( M[3, y] + 8 = 19; \ M[3, x] + 5 = 1; \ M[3, z] + (-4) = -2. \)
For x, check \( M[3, t] + (-2) = -4. \)
For y, check \( M[3, x] + (-3) = -7; \ M[3, z] + 9 = 11. \)
For z, check \( M[3, x] + 4 = 0; \ M[3, s] + 2 = 2. \)

\[
\begin{array}{cccccc}
M & 0 & 1 & 2 & 3 & 4 \\
\hline
s & 0 & 0 & 0 & 0 & 0 \\
t & \infty & \infty & \infty & 2 & 2 \\
x & \infty & \infty & \infty & 4 & 4 \\
y & \infty & \infty & 11 & 11 & -7 \\
z & \infty & 2 & 2 & 2 & 0 \\
\end{array}
\]

From the above table we can find that the shortest path from t to s is: t → z → x → t → z → x → t → z → x → t → z → x → ... , which is a negative cycle (t → z → x), so there is no shortest path in this graph.
**Exercise 24.3-1**

Run Bellman-Ford algorithm on the directed graph below, first using vertex $s$ as the destination and then using vertex $z$ as the destination.

![Graph Image]

**Solution:**

[Here we provide only the tables and refer you back to the previous solution for help with details of the logic.]

(a) Use vertex $s$ as the destination

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
 & 0 & 1 & 2 & 3 & 4 \\
\hline
s & 0 & 0 & 0 & 0 & 0 \\
\hline
t & \infty & \infty & \infty & \infty & 11 \\
\hline
x & \infty & \infty & 5 & 5 & 5 \\
\hline
y & \infty & \infty & 9 & 9 & 9 \\
\hline
z & \infty & 3 & 3 & 3 & 3 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
 & 0 & 1 & 2 & 3 & 4 & first \\
\hline
s & NIL & NIL & NIL & NIL & NIL & NIL \\
\hline
t & NIL & NIL & NIL & x & x & x \\
\hline
x & NIL & NIL & z & z & z & z \\
\hline
y & NIL & NIL & z & z & z & z \\
\hline
z & NIL & s & s & s & s & s \\
\hline
\end{array}
\]

(b) Use vertex $z$ as the destination

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
 & 0 & 1 & 2 & 3 & 4 \\
\hline
z & 0 & 0 & 0 & 0 & 0 \\
\hline
s & \infty & \infty & \infty & 11 & 11 \\
\hline
t & \infty & \infty & 8 & 8 & 8 \\
\hline
x & \infty & 2 & 2 & 2 & 2 \\
\hline
y & \infty & 6 & 6 & 6 & 6 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
 & 0 & 1 & 2 & 3 & 4 & first \\
\hline
z & NIL & NIL & NIL & NIL & NIL & NIL \\
\hline
s & NIL & NIL & y & y & y & y \\
\hline
t & NIL & NIL & x & x & x & x \\
\hline
x & NIL & z & z & z & z & z \\
\hline
y & NIL & z & z & z & z & z \\
\hline
\end{array}
\]